THE STABILIZER SETS OF 5-QUBIT STABILIZER CODE

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ABSTRACT

Background: when quantum information is transmitted or manipulated in noisy environments, the information is lost gradually because of adverse interaction with the environment. To protect the fragile quantum states, error-correction codes are needed to protect the quantum data during the processes of quantum computation and communication. **Objectives:** the present study were designed to investigate the quantum errors correcting codes (QEEC), precisely quantum stabilizer codes. **Methods:** we introduce the formalism of stabilizer and we obtain such results the codewords of the quantum stabilizer code. **Results,** as results it has been found that the 5 qubit code has different stabilizer groups. **Conclusions:** the classical and specially the stabilizer quantum codes have much in common.

Keywords: Additive quantum codes, Stabilizer groups.

1. INTRODUCTION

Currently, quantum computer [1, 2, 3, 4, 5, 6] is supposed to work with qubits, the smallest unit of information. This information is represented by some physical quantities susceptibility to errors dramatically changes in comparison to classical implementation of the bit of information [7]. Theoretically repeated frequent measurement of the classical register can kick the bit back to the nearer of 0 or 1. However the same technique cannot be used in a quantum computer, since measuring the qubit would break down the superposition between the logical zero and the logical one.

In addition, small interactions between qubits and the environment is a kind of continuous measurement of the system and more interactions increases more information lost and thebegin to look like classical system. This is for this reason (decoherence) we use the stabilizer codes.

In this paper, the objective is the study of additive codes (also called stabilizer codes) which have been an important class of codes for most applications. In section 2 we give an introduction of quantum error correcting code and the properties of this quantum codes. In section 3 we demonstrate how data encoded in a five-qubit quantum error correction and the analysis will be extended to a different stabilizers. Finally a conclusion is given in section 4.

2. MATERIALS AND METHODS

2.1 Quantum error correction codes

A quantum error can be any 2×2 unitary matrix. It comes natural then to look at set of Pauli matrices as they span the space on 2×2 unitary matrices. They form a group with multiplicative factors ±1, $\pm i$ called Pauli group P_1 since: X, Y, and Z anti-commute, i.e. $XZ = -ZX$, and similarly $\{X, Y\} = 0$ and $\{Y, Z\} = 0$.

$$P_1 = \{\pm I, \pm iI, \pm X, \pm iX, \pm Y, \pm iY, \pm Z, \pm iZ\} \qquad (1)$$

Thus, the n-qubit Pauli group P_n consists of the 4^n tensor products of I, X, Y, and Z, and an overall phase of ±1, or ±i, for a total of 4^{n+1} elements.

$$P_n = \{g \,/\, g = g_1 \otimes g_2 \otimes \ldots \otimes g_n,\ \ g_i \in \{I, X, Y, Z\}\}. \qquad (2)$$

An important property of Pauli group that it is not an Abelian group. Another algebraic property that is quite useful is that any pair of elements of P_n either commute or anti-commute. Also, the square of any element of P_n is ±1. We shall only need to work with the elements with square +1, which are tensor products of I, X, Y, and Z with an overall sign ±1; the phase i is only necessary to make P_n a group. We can now define the weight of an operator in P_n to be the number of tensor factors which are not I.

2.2 Error correction Model

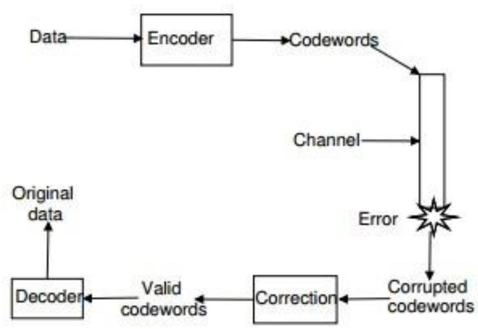

Figure 1: The figure presents the quantum error correction model.

1- Information to be transferred.
2- We encode the information into codewords.
3- These codewords go through the channel.
4- We will get the corrupted codewords.
5- The quantum error correcting code can be correct these corrupted codewords.
6- Decoding to obtain the initial information.

2.3 Properties of a stabilizer

1-The set of generators of the stabilizer [8, 9, 10, 11, 12, 13], forms a group under the multiplication:

$$\text{If } M|\psi\rangle = |\psi\rangle \text{ and } N|\psi\rangle = |\psi\rangle, \text{ then } MN|\psi\rangle = |\psi\rangle. \qquad (3)$$

2-Any stabilizer subgroup of the Pauli group corresponding to a non-trivial stabilizer code, is Abelian:

$$\text{If } M|\psi\rangle = |\psi\rangle \text{ and } N|\psi\rangle = |\psi\rangle, \qquad (4)$$

$$\text{then } (MN - NM)|\psi\rangle = MN|\psi\rangle - NM|\psi\rangle = 0. \qquad (5)$$

$$\text{For Pauli matrices} \Rightarrow\ MN = NM. \qquad (6)$$

3-Given any Abelian group S of Pauli operators, define a code space

$$C(S) = \{|\psi\rangle,\ \ M|\psi\rangle = |\psi\rangle\ \ \forall\ M \in S\}, \qquad (7)$$

then C(S) encodes k logical qubits in n physical qubits when S has n−k generators (so size 2^{n-k}).

4-Let $\{M_1, M_2, \ldots, M_{n-k}\}$ are the generators of the stabilizer S, where $M_i \in P_n$ $(1 \le i \le n-k)$, H is an $(n-k)\times 2n$ matrix over GF(2) whose rows contains the vectors $\phi(M_1), \phi(M_2), \ldots, \phi(M_{n-k})$ that is

$$H = \begin{bmatrix} \phi(M_1) \\ \dots \\ \phi(M_{n-k}) \end{bmatrix} = [H_X, H_Z], \quad (8)$$

H is called the check matrix of S.

3. RESULTS AND DISCUSSION: The five qubit stabilizer code

Additive quantum codes, also known as stabilizer codes utilize some simple structural properties of the space of possible errors [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. They essentially emerge in a attempt to generalize the techniques from the theory of classical error correction to the quantum domain.

The [[n, k, d]] = [[5, 1, 3]] perfect code encodes a single qubit (k = 1), and corrects all errors of weight 1 (since (d − 1)/2 = 1). This code is the smallest single-error correcting quantum code.

We concentrate our intention here on the stabilizer sets, where, the number of stabilizer sets in the Hilbert space C^{2^n} that maintain a +1 eigenspace of dimension 2^k is given by

$$2^n \prod_{k=0}^{n-1}(2^{n-k} - 1). \quad (9)$$

The cardinality of [[5, 1, 3]] is $|S| = 2^{n-k} = 16$ and the set $D_S^{[[5,1,3]]}$ of n − k = 4 stabilizer group generators is given by

$$D_S^{[[5,1,3]]} = < \sigma_x\sigma_z\sigma_z\sigma_x I, I\sigma_x\sigma_z\sigma_z\sigma_x, \sigma_x I\sigma_x\sigma_z\sigma_z, \sigma_z\sigma_x I\sigma_x\sigma_z >. \quad (10)$$

The code is the simultaneous eigenspace with eigenvalue 1 of 4 commuting check operators (stabilizer generators)

$$\begin{aligned} N_1 &= \sigma_x\sigma_z\sigma_z\sigma_x I, \\ N_2 &= I\sigma_x\sigma_z\sigma_z\sigma_x, \\ N_3 &= \sigma_x I\sigma_x\sigma_z\sigma_z, \\ N_4 &= \sigma_z\sigma_x I\sigma_x\sigma_z. \end{aligned} \quad (11)$$

All of these stabilizer generators square to I, they are mutually commuting because there are two collisions between σ_x and σ_z. The other three generators are obtained from the first by cyclic permutations.

Table1: The table presents the generator which anti-commute with the error.

Error operator	Generator anti-commute with the error
$E_1 = \sigma_x \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I$	$N_4 = \sigma_z\sigma_x I\sigma_x\sigma_z$
$E_2 = \sigma_I \otimes \sigma_x \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I$	$N_1 = \sigma_x\sigma_z\sigma_z\sigma_x I$
$E_3 = \sigma_I \otimes \sigma_I \otimes \sigma_x \otimes \sigma_I \otimes \sigma_I$	$N_1 = \sigma_x\sigma_z\sigma_z\sigma_x I$, $N_2 = I\sigma_x\sigma_z\sigma_z\sigma_x\sigma_x$
$E_4 = \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_x \otimes \sigma_I$	$N_2 = I\sigma_x\sigma_z\sigma_z\sigma_x\sigma_x$, $N_3 = \sigma_x I\sigma_x\sigma_z\sigma_z$
$E_5 = \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_x$	$N_3 = \sigma_x I\sigma_x\sigma_z\sigma_z$, $N_4 = \sigma_z\sigma_x I\sigma_x\sigma_z$
$E_6 = \sigma_z \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I$	$N_1 = \sigma_x\sigma_z\sigma_z\sigma_x I$, $N_3 = \sigma_x I\sigma_x\sigma_z\sigma_z$
$E_7 = \sigma_I \otimes \sigma_z \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I$	$N_2 = I\sigma_x\sigma_z\sigma_z\sigma_x\sigma_x$, $N_4 = \sigma_z\sigma_x I\sigma_x\sigma_z$
$E_8 = \sigma_I \otimes \sigma_I \otimes \sigma_z \otimes \sigma_I \otimes \sigma_I$	$N_3 = \sigma_x I\sigma_x\sigma_z\sigma_z$
$E_9 = \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_z \otimes \sigma_I$	$N_1 = \sigma_x\sigma_z\sigma_z\sigma_x I$, $N_4 = \sigma_z\sigma_x I\sigma_x\sigma_z$
$E_{10} = \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_z$	$N_2 = I\sigma_x\sigma_z\sigma_z\sigma_x\sigma_x$

Table 2: The table presents the commutation and anti-commutation.

	N_1	N_2	N_3	N_4
X_1	+	+	+	−
Y_1	−	+	−	−
Z_1	−	+	−	+
X_2	−	+	+	+
Y_2	−	−	+	−
Z_2	+	−	+	−
X_3	−	−	+	+
Y_3	−	−	−	+
Z_3	+	+	−	+
X_4	+	−	−	+
Y_4	−	−	−	−
Z_4	−	+	+	−
X_5	+	+	−	−
Y_5	+	−	−	−
Z_5	+	−	+	+
I	+	+	+	+

The stabilizer and generator are given by

$$\begin{pmatrix} \sigma_x\sigma_z\sigma_z\sigma_x I \\ I\sigma_x\sigma_z\sigma_z\sigma_x \\ \sigma_x I\sigma_x\sigma_z\sigma_z \\ \sigma_z\sigma_x I\sigma_x\sigma_z \\ \sigma_x\sigma_x\sigma_x\sigma_x\sigma_x \\ \sigma_z\sigma_z\sigma_z\sigma_z\sigma_z \end{pmatrix}, \tag{12}$$

and the check parity matrix is

$$\mathrm{H} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & | & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & & 1 & 0 & 1 & 0 & 0 \end{pmatrix}. \tag{13}$$

We can take the basis codewords for this code to be

$$|0_L>= \sum_{N\epsilon D_S} N|00000>, \tag{14}$$

and

$$|1_L>= \overline{X}|0_L>, \tag{15}$$

that is

$$|0_L>=\frac{1}{4}[|00000>+|10010>+|01001>+|01010>-|11011>- |00100>-|11000>-|11101>-|00011>-|11110>- |01111>-|10001>-|01100>+|10111>+|00101> +|10100>], \tag{16}$$

and

$$|1_L>=\frac{1}{4}[|11111>+|01101>+|10110>+|01011>+|10101>- |00100>-|11001>-|00111>-|00010>-|11100>- |00001>-|10000>-|01110>-|10011>-|01000> +|11010>]. \tag{17}$$

We can take the stabilizer

$$B_S^{[[5,1,3]]} = <\sigma_x\sigma_x\sigma_z I\sigma_z, \sigma_z\sigma_x\sigma_x\sigma_z I, I\sigma_z\sigma_x\sigma_x\sigma_z, \sigma_z I\sigma_z\sigma_x\sigma_x>, \tag{18}$$

Where $B_S^{[[5,1,3]]}$ is the dual of $D_S^{[[5,1,3]]}$. The code is the simultaneous eigenspace with eigenvalue 1 of 4 commuting check operators (stabilizer generators)

$$\begin{aligned} M_1 &= \sigma_x\sigma_x\sigma_z I\sigma_z, \\ M_2 &= \sigma_z\sigma_x\sigma_x\sigma_z I, \\ M_3 &= I\sigma_z\sigma_x\sigma_x\sigma_z, \\ M_4 &= \sigma_z I\sigma_z\sigma_x\sigma_x. \end{aligned} \tag{19}$$

Table 3: The table presents the generator which anti-commute with the error.

Error operator	Generator anti-commute with the error
$E_1 = \sigma_x\otimes\sigma_I\otimes\sigma_I\otimes\sigma_I\otimes\sigma_I$	$M_2 = \sigma_z\sigma_x\sigma_x\sigma_z I$, $M_4 = \sigma_z I\sigma_z\sigma_x\sigma_x$
$E_2 = \sigma_I\otimes\sigma_x\otimes\sigma_I\otimes\sigma_I\otimes\sigma_I$	$M_3 = I\sigma_z\sigma_x\sigma_x\sigma_z$
$E_3 = \sigma_I\otimes\sigma_I\otimes\sigma_x\otimes\sigma_I\otimes\sigma_I$	$M_1 = \sigma_x\sigma_x\sigma_z I\sigma_z$, $M_4 = \sigma_z I\sigma_z\sigma_x\sigma_x$
$E_4 = \sigma_I\otimes\sigma_I\otimes\sigma_I\otimes\sigma_x\otimes\sigma_I$	$M_2 = \sigma_z\sigma_x\sigma_x\sigma_z I$
$E_5 = \sigma_I\otimes\sigma_I\otimes\sigma_I\otimes\sigma_I\otimes\sigma_x$	$M_1 = \sigma_x\sigma_x\sigma_z I\sigma_z$, $M_3 = I\sigma_z\sigma_x\sigma_x\sigma_z$
$E_6 = \sigma_z\otimes\sigma_I\otimes\sigma_I\otimes\sigma_I\otimes\sigma_I$	$M_1 = \sigma_x\sigma_x\sigma_z I\sigma_z$
$E_7 = \sigma_I\otimes\sigma_z\otimes\sigma_I\otimes\sigma_I\otimes\sigma_I$	$M_1 = \sigma_x\sigma_x\sigma_z I\sigma_z$, $M_2 = \sigma_z\sigma_x\sigma_x\sigma_z I$
$E_8 = \sigma_I\otimes\sigma_I\otimes\sigma_z\otimes\sigma_I\otimes\sigma_I$	$M_2 = \sigma_z\sigma_x\sigma_x\sigma_z I$, $M_3 = I\sigma_z\sigma_x\sigma_x\sigma_z$
$E_9 = \sigma_I\otimes\sigma_I\otimes\sigma_I\otimes\sigma_z\otimes\sigma_I$	$M_3 = I\sigma_z\sigma_x\sigma_x\sigma_z$, $M_4 = \sigma_z I\sigma_z\sigma_x\sigma_x$
$E_{10} = \sigma_I\otimes\sigma_I\otimes\sigma_I\otimes\sigma_I\otimes\sigma_z$	$M_4 = \sigma_z I\sigma_z\sigma_x\sigma_x$

Table 4: The table presents the commutation and anti-commutation.

	M_1	M_2	M_3	M_4
X_1 Y_1 Z_1	+ − −	− − +	+ + +	− − +
X_2 Y_2 Z_2	+ − −	+ − −	− − +	+ + +
X_3 Y_3 Z_3	− − +	+ − −	+ − −	− − +
X_4 Y_4 Z_4	+ + +	− − +	+ − −	+ − −
X_5 Y_5 Z_5	− − +	+ + +	− − +	+ − −
I	+	+	+	+

The stabilizer and generator are given by

$$\begin{pmatrix} \sigma_x\sigma_x\sigma_z I\sigma_z \\ \sigma_z\sigma_x\sigma_x\sigma_z I \\ I\sigma_z\sigma_x\sigma_x\sigma_z \\ \sigma_z I\sigma_z\sigma_x\sigma_x \\ II\sigma_z\sigma_y\sigma_z \\ II\sigma_x\sigma_z\sigma_x \end{pmatrix}, \tag{20}$$

and the check parity matrix is

$$\mathrm{H} = \left(\begin{matrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{matrix} \;\middle|\; \begin{matrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}\right). \tag{21}$$

We can take the basis codewords for this code to be

$$|0_L>= \frac{1}{4}[-|00000> + |10010> + |01001> + |01010> + |11011> - |00110> - |11000> + |11101> - |00011> + |11110> + |01111> - |10001> - |01100> - |10111> + |00101> - |10100>], \tag{22}$$

and

$$|1_L>= \frac{1}{4}[|11111> + |01101> - |10110> - |01011> - |10101> - |00100> + |11001> + |00111> - |00010> + |11100> - |00001> - |10000> - |01110> - |10011> - |01000> - |11010>]. \tag{23}$$

We can take the stabilizer

$$G_S^{[[5,1,3]]} =< I\sigma_z\sigma_y\sigma_y\sigma_z, I\sigma_x\sigma_z\sigma_z\sigma_x, \sigma_x I\sigma_x\sigma_z\sigma_z, \sigma_z I\sigma_z\sigma_y\sigma_y >, \tag{24}$$

where

$$\begin{aligned} g_1 &= I\sigma_z\sigma_y\sigma_y\sigma_z, \\ g_2 &= I\sigma_x\sigma_z\sigma_z\sigma_x, \\ g_3 &= \sigma_x I\sigma_x\sigma_z\sigma_z, \\ g_4 &= \sigma_z I\sigma_z\sigma_y\sigma_y \end{aligned} \tag{25}$$

Table 5: The table presents the generator which anti-commute with the error.

Error operator	Generator anti-commute with the error
$E_1 = \sigma_x \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I$	$g_4 = \sigma_z I \sigma_z \sigma_y \sigma_y$
$E_2 = \sigma_I \otimes \sigma_x \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I$	$g_1 = I \sigma_z \sigma_y \sigma_y \sigma_z$
$E_3 = \sigma_I \otimes \sigma_I \otimes \sigma_x \otimes \sigma_I \otimes \sigma_I$	$g_1 = I\sigma_z\sigma_y\sigma_y\sigma_z$, $g_2 = I\sigma_x\sigma_z\sigma_z\sigma_x$, $g_4 = \sigma_z I \sigma_z \sigma_y \sigma_y$
$E_4 = \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_x \otimes \sigma_I$	$g_1 = I\sigma_z\sigma_y\sigma_y\sigma_z$, $g_2 = I\sigma_x\sigma_z\sigma_z\sigma_x$, $g_3 = \sigma_x I \sigma_x \sigma_z \sigma_z$
$E_5 = \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_x$	$g_1 = I\sigma_z\sigma_y\sigma_y\sigma_z$, $g_3 = \sigma_x I \sigma_x \sigma_z \sigma_z$, $g_4 = \sigma_z I \sigma_z \sigma_y \sigma_y$
$E_6 = \sigma_z \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I$	$g_3 = \sigma_x I \sigma_x \sigma_z \sigma_z$
$E_7 = \sigma_I \otimes \sigma_z \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I$	$g_2 = I\sigma_x\sigma_z\sigma_z\sigma_x$
$E_8 = \sigma_I \otimes \sigma_I \otimes \sigma_z \otimes \sigma_I \otimes \sigma_I$	$g_1 = I\sigma_z\sigma_y\sigma_y\sigma_z$, $g_3 = \sigma_x I \sigma_x \sigma_z \sigma_z$
$E_9 = \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_z \otimes \sigma_I$	$g_1 = I\sigma_z\sigma_y\sigma_y\sigma_z$, $g_4 = \sigma_z I \sigma_z \sigma_y \sigma_y$
$E_{10} = \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_z$	$g_2 = I\sigma_x\sigma_z\sigma_z\sigma_x$, $g_4 = \sigma_z I \sigma_z \sigma_y \sigma_y$
$E_{11} = \sigma_y \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I$	$g_3 = \sigma_x I \sigma_x \sigma_z \sigma_z$, $g_4 = \sigma_z I \sigma_z \sigma_y \sigma_y$
$E_{12} = \sigma_I \otimes \sigma_y \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I$	$g_1 = I\sigma_z\sigma_y\sigma_y\sigma_z$, $g_2 = I\sigma_x\sigma_z\sigma_z\sigma_x$
$E_{13} = \sigma_I \otimes \sigma_I \otimes \sigma_y \otimes \sigma_I \otimes \sigma_I$	$g_2 = I\sigma_x\sigma_z\sigma_z\sigma_x$, $g_3 = \sigma_x I \sigma_x \sigma_z \sigma_z$
$E_{14} = \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_y \otimes \sigma_I$	$g_2 = I\sigma_x\sigma_z\sigma_z\sigma_x$, $g_3 = \sigma_x I \sigma_x \sigma_z \sigma_z$
$E_{15} = \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_y$	$g_1 = I\sigma_z\sigma_y\sigma_y\sigma_z$, $g_2 = I\sigma_x\sigma_z\sigma_z\sigma_x$, $g_3 = \sigma_x I \sigma_x \sigma_z \sigma_z$

Table 6: The table presents the commutation and anti-commutation.

	g_1	g_2	g_3	g_4
X_1 Y_1 Z_1	+ + +	+ + +	+ – –	– – +
X_2 Y_2 Z_2	– – +	+ – –	+ + +	+ + +
X_3 Y_3 Z_3	– + –	– – +	+ – –	– – +
X_4 Y_4 Z_4	– + –	– – +	– – +	+ – –
X_5 Y_5 Z_5	– – +	+ – –	– – +	– + –
I	+	+	+	+

The stabilizer and generator are given by

$$\begin{pmatrix} I\sigma_z\sigma_y\sigma_y\sigma_z \\ I\sigma_x\sigma_z\sigma_z\sigma_x \\ \sigma_x I\sigma_x\sigma_z\sigma_z \\ \sigma_z I\sigma_z\sigma_y\sigma_y \\ \sigma_x\sigma_x\sigma_x\sigma_x\sigma_x \\ \sigma_z\sigma_z\sigma_z\sigma_z\sigma_z \end{pmatrix}, \tag{26}$$

and the check parity matrix is

$$\mathrm{H} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & | & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & | & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & | & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & | & 0 & 0 & 0 & 1 & 1 \end{pmatrix}. \tag{27}$$

We can take the basis codewords for this code to be

$$\begin{aligned} |0_L>= \frac{1}{4}[&|00000>+|10010>+|01001>+|01010>+|11011>+ \\ &|00110>-|11000>-|11101>+|00011>-|11110>+ \\ &|01111>-|10001>-|01100>+|10111>+|00101> \\ &+|10100>], \end{aligned} \tag{28}$$

and

$$|1_L>=\overline{X}|0_L>. \tag{29}$$

We can take the stabilizer

$$L_S^{[[5,1,3]]} =< \sigma_y\sigma_y\sigma_z I\sigma_z, \sigma_z\sigma_y\sigma_y\sigma_z I, I\sigma_z\sigma_y\sigma_y\sigma_z, \sigma_z I\sigma_z\sigma_y\sigma_y >, \quad (30)$$

where

$$\begin{aligned} l_1 &= \sigma_y\sigma_y\sigma_z I\sigma_z, \\ l_2 &= \sigma_z\sigma_y\sigma_y\sigma_z I, \\ l_3 &= I\sigma_z\sigma_y\sigma_y\sigma_z, \\ l_4 &= \sigma_z I\sigma_z\sigma_y\sigma_y. \end{aligned} \quad (31)$$

Table 7: The table presents the generator which anti-commute with the error.

or operator	Generator anti-commute with the error
E1 = $\sigma_x \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I$	$l_1 = \sigma_y\sigma_y\sigma_z I\sigma_z,\ l_2 = \sigma_z\sigma_y\sigma_y\sigma_z I, l_4 = \sigma_z I\sigma_z\sigma_y\sigma_y$
E2 = $\sigma_I \otimes \sigma_x \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I$	$l_1 = \sigma_y\sigma_y\sigma_z I\sigma_z, l_2 = \sigma_z\sigma_y\sigma_y\sigma_z I,\ l_3 = I\sigma_z\sigma_y\sigma_y\sigma_z$
E3 = $\sigma_I \otimes \sigma_I \otimes \sigma_x \otimes \sigma_I \otimes \sigma_I$	$l_1 = \sigma_y\sigma_y\sigma_z I\sigma_z,\ l_2 = \sigma_z\sigma_y\sigma_y\sigma_z I,\ l_3 = I\sigma_z\sigma_y\sigma_y\sigma_z,\ l_4 = \sigma_z I\sigma_z\sigma_y\sigma_y$
E4 = $\sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_x \otimes \sigma_I$	$l_2 = \sigma_z\sigma_y\sigma_y\sigma_z I,\ l_3 = I\sigma_z\sigma_y\sigma_y\sigma_z,\ l_4 = \sigma_z I\sigma_z\sigma_y\sigma_y$
E5 = $\sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_x$	$l_1 = \sigma_y\sigma_y\sigma_z I\sigma_z,\ l_3 = I\sigma_z\sigma_y\sigma_y\sigma_z,\ l_4 = \sigma_z I\sigma_z\sigma_y\sigma_y$
E6 = $\sigma_z \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I$	$l_1 = \sigma_y\sigma_y\sigma_z I\sigma_z$
E7 = $\sigma_I \otimes \sigma_z \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I$	$l_1 = \sigma_y\sigma_y\sigma_z I\sigma_z, l_2 = \sigma_z\sigma_y\sigma_y\sigma_z I$
E8 = $\sigma_I \otimes \sigma_I \otimes \sigma_z \otimes \sigma_I \otimes \sigma_I$	$l_2 = \sigma_z\sigma_y\sigma_y\sigma_z I,\ l_3 = I\sigma_z\sigma_y\sigma_y\sigma_z$
E9 = $\sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_z \otimes \sigma_I$	$l_3 = I\sigma_z\sigma_y\sigma_y\sigma_z,\ l_4 = \sigma_z I\sigma_z\sigma_y\sigma_y$
E10 = $\sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_z$	$l_4 = \sigma_z I\sigma_z\sigma_y\sigma_y$
E11 = $\sigma_y \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I$	$l_2 = \sigma_z\sigma_y\sigma_y\sigma_z I,\ l_4 = \sigma_z I\sigma_z\sigma_y\sigma_y$
E12 = $\sigma_I \otimes \sigma_y \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I$	$l_2 = \sigma_z\sigma_y\sigma_y\sigma_z I$
E13 = $\sigma_I \otimes \sigma_I \otimes \sigma_y \otimes \sigma_I \otimes \sigma_I$	$l_1 = \sigma_y\sigma_y\sigma_z I\sigma_z,\ l_4 = \sigma_z I\sigma_z\sigma_y\sigma_y$
E14 = $\sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_y \otimes \sigma_I$	$l_2 = \sigma_z\sigma_y\sigma_y\sigma_z I$
E15 = $\sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_I \otimes \sigma_y$	$l_1 = \sigma_y\sigma_y\sigma_z I\sigma_z,\ l_3 = I\sigma_z\sigma_y\sigma_y\sigma_z$

Table 8: The table presents the commutation and anti-commutation.

	l_1	l_2	l_3	l_4
X_1	−	−	+	−
Y_1	+	−	+	−
Z_1	−	+	+	+
X_2	−	−	−	+
Y_2	+	+	−	+
Z_2	−	−	+	+
X_3	−	−	−	−
Y_3	−	+	+	−
Z_3	+	−	−	+
X_4	+	−	−	−
Y_4	+	−	+	+
Z_4	+	+	−	−
X_5	−	+	−	−
Y_5	−	−	−	+
Z_5	+	−	+	+
I	+	+	+	+

The stabilizer and generator are given by

$$\begin{pmatrix} \sigma_y\sigma_y\sigma_z I\sigma_z \\ \sigma_z\sigma_y\sigma_y\sigma_z I \\ I\sigma_z\sigma_y\sigma_y\sigma_z \\ \sigma_z I\sigma_z\sigma_y\sigma_y \\ \sigma_x\sigma_x\sigma_x\sigma_x\sigma_x \\ \sigma_z\sigma_z\sigma_z\sigma_z\sigma_z \end{pmatrix}, \tag{32}$$

and the check parity matrix is

$$\mathrm{H} = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & | & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & | & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & | & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & | & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{33}$$

We can take the basis codewords for this code to be

$$|0_L >= \textstyle\sum_{N\epsilon D_S} l|00000 >, \tag{34}$$

and

$$|1_L >= \overline{X}|0_L >. \tag{35}$$

We can take now the stabilizer

$$V_S^{[[5,1,3]]} =< \sigma_y\sigma_z\sigma_z\sigma_y I, I\sigma_y\sigma_z\sigma_z\sigma_y, \sigma_y I\sigma_y\sigma_z\sigma_z, \sigma_z\sigma_y I\sigma_y\sigma_z >, \tag{36}$$

where

$$\begin{aligned} v_1 &= \sigma_y\sigma_z\sigma_z\sigma_y I, \\ v_2 &= I\sigma_y\sigma_z\sigma_z\sigma_y, \\ v_3 &= \sigma_y I\sigma_y\sigma_z\sigma_z, \\ v_4 &= \sigma_z\sigma_y I\sigma_y\sigma_z. \end{aligned} \tag{37}$$

The stabilizer and generator are given by

$$\begin{pmatrix} \sigma_y\sigma_z\sigma_z\sigma_y I \\ I\sigma_y\sigma_z\sigma_z\sigma_y \\ \sigma_y I\sigma_y\sigma_z\sigma_z \\ \sigma_z\sigma_y I\sigma_y\sigma_z \\ \sigma_x\sigma_x\sigma_x\sigma_x\sigma_x \\ \sigma_z\sigma_z\sigma_z\sigma_z\sigma_z \end{pmatrix}, \tag{38}$$

and the check parity matrix is

$$\mathrm{H} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & | & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & | & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & | & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & | & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{39}$$

We can take the basis code words for this code to be

$$|0_L >= \sum_{N\epsilon D_S} v|00000 >, \tag{40}$$

and

$$|1_L >= \bar{X}|0_L >. \tag{41}$$

4. CONCLUSION

For a quantum computer, the classical error-correction techniques cannot be directly carried, because the classical techniques assume we can measure all of the bits in the computer. This is different in the quantum world, this would destroy any entanglement between qubits, since the quantum computer needs to keep phase information in entangled states conversely to a classical computer only needs to preserve the bit values of 0 and 1. Thus, while quantum error-correcting codes are related to classical codes, they require a somewhat new approach.

The class of quantum codes called stabilizer codes considered as a most good and effective class of quantum codes for the stabilization of the state quantum. This is given by a formalism of the stabilizer group.

In this paper we are finding that we can stabilize the subspace of the quantum code by different stabilizer sets not a unique stabilizer set. But in generally the no stabilizer codes are more effective compared to the stabilizer codes.

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