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CLASSICAL ERRORS CORRECTING CODES AND QUANTUM ERRORS CORRECTING CODES

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ABSTRACT

Background: Quantum error correction (*QEC*) comes from the marriage of quantum mechanics with the classical theory of error correcting codes. Error correction is a central concept in classical information theory, and quantum error correction is similarly foundational in quantum information theory. Both are concerned with the fundamental problem of communication, and/or information storage, in the presence of noise. **Objectives:** the present study was designed to investigate both the classical error correcting codes and a quantum error correcting codes. **Methods:** we study the main concepts of code theory such as, the distance of the code, the weight. We concentrated our work on both the stabilizer quantum codes and no stabilizer quantum codes. We give the correspondence of each concept between the classical and the quantum codes, also between the stabilizer quantum codes and CWS codes. **Results:** as results it has been found that the classical and specially the stabilizer quantum codes have much in common. These codes are treated in the same way specially the distance, the weight, the stabilizer and the syndrome. **Conclusions:** the CWS codes which contains the stabilizer codes and non-stabilizer codes is more effective and more efficient compared to the stabilizer codes.
Keywords: *Qubit, Stabilizer codes, CWS codes.*

1. INTRODUCTION

Coding theory [1] is a branch of mathematics which seeks optimal solutions to problems concerning the safe and accurate transfer of information. Shannon (1948) established the topic of coding theory with his seminal paper on the mathematics of communication. Hamming (1950) then introduced the concept of an error correcting codes with his work on the correction of errors on magnetic storage media [1,2,3].
In recent years, coding theory has evolved beyond its original classical setting and is considered within a quantum theoretical perspective.

Quantum computation [4,5,6] has attracted great interest because efficient algorithms have been found to solve a variety of classical problems, such as factoring, that are believed to be hard for classical computers. Moreover, as the processor size in classical computers continues to scale down, the quantum nature of the components of classical computers will begin to be important. Performing reliable classical computations on machines built of quantum components is an important problem; the possibility of exploiting quantum effects to achieve remarkable new performance is an even greater incentive to understand these systems.

In quantum computation [6,7,8,9,10,11,12], it is important to preserve coherence of quantum information. For this purpose, quantum information must be protected by quantum error correcting codes (*QECCs*) [4,5,6,7,8,9,13,14,15,16] from unwanted interactions and quantum noise. While there are many classical error correction schemes which perform close to the classical channel capacity, it is hard to apply classical error correction schemes directly to *QEC* because of various properties of quantum system such as no-cloning, continuous error models, and measurement-disturbance tradeoffs which do not exist in classical systems. Despite these differences between classical error correction and *QEC*, it is still possible to develop quantum error correcting codes based on classical error-correcting structure Stabilizer codes, were introduced independently by Gottesman (1997) and Calderbank (1998), are analogues of classical additive codes. This type of code is specified by a stabilizer group, which is an Abelian subgroup of the Pauli group on n qubits. The code space of a stabilizer code is fixed by this stabilizer group. That is, it is a joint eigenspace with eigenvalue 1. Stabilizer codes can be constructed from classical linear codes that satisfy a particular dual-containing constraint.

Recently, a more general framework, codeword stabilized quantum (CWS) codes [15,16,17], which includes both additive and non-additive quantum codes. CWS codes in standard form are defined by a graph [15] and a classical binary code. An important aspect of the CWS framework is the fact that any Pauli error is equivalent in its effects to an error consisting only of Z operators. This means that any Pauli error can be treated as a classical binary error. Using a set of these induced errors as the desired correctable error set, a quantum error correcting code can be constructed from a corresponding classical binary code, albeit one with a nonstandard error set.

The set of codes that can be expressed in this way includes the stabilizer codes, but many others as well. These additional CWS codes are non-additive. Non additive codes (in principle) can encode a logical state of higher dimension than a stabilizer code with the same length in physical qubits, while protecting it from same number of errors. This promises potential gains in performance for quantum error correction. (Note, however, that none of the non-additive codes discovered so far have a minimum distance greater than three.)

In the second section of this paper, we talk about the important postulates of quantum mechanics. We will discuss in section 3 the quantum (stabilizer) and classical error correcting codes and the correspondence between them which is the main objective. In the fourth section we give some examples of quantum codes. In section 5 the analysis will be extended to CWS codes and the correspondence between stabilizer codes and CWS codes is discussed section 6. Finally a conclusion is given in section 7.

2. MATERIALS AND METHODS

2.1 The quantum postulates:

Postulate I:
Each physical system is represented by a Hilbert space and described by physical quantities and state represented by linear operators in that space.

Postulate II:
Each physical quantity of a quantum system is represented by a positive Hermitian operator O, the expectation value of which is given by $tr(\rho O)$, where ρ is the bounded positive Hermitian trace-class operator representing the state of the system.

Postulate III:
When a physical quantity of a system initially prepared in a state represented by the statistical operator ρ is measured, the state of the system immediately after this measurement is represented by the statistical operator

$$\rho' = \frac{P_k \rho P_k}{tr(\rho P_k)}, \qquad (1)$$

where P_k is the projection operator onto the subspace corresponding to measurement outcome k, with a probability given by the expectation value of P_k for ρ. This postulate is essential for connecting the behavior of quantum systems with that of the classical systems used to measure them.

Postulate IV:
Each physical system composed of two or more subsystems is represented by the Hilbert space that is the tensor product of the Hilbert spaces representing its subsystems; the operators representing its physical quantities act in this product space.

Postulate V:
The time evolution of the state of each closed physical system, that is, each physical system not interacting with anything outside of itself, takes place according to

$$\rho(t)=U(t)\rho(0)U^{\dagger}(t), \qquad (2)$$

where t is the time parameter and $U = e^{-tH/\hbar}$ is a unitary operator, H being the generator of time translations.

3. RESULTS AND DISCUSSION

Table 1: The table presents the all results about the correspondence between the classical and stabilizer quantum codes.

ECC	*QECC*
Classical information theory: Is the common name for the information theory of Shannon, which is a probabilistic theory to quantify the average information content of a set of messages, including computer coding satisfies a precise statistical distribution.	**Quantum information theory:** New way to process information, using the quantum mechanical properties. This is not tied to a particular system: Any quantum system with some specific property taken can be chosen.
Classic computer: Is based on the existence of the notion bit, a discrete variable can take two contrasting states, denoted 0 and 1.	**Quantum computer:** A quantum computer would handle instead of classical bits, quantum bits, called "qubits". These are the simplest quantum systems, two level systems. A qubit can be either in the state $\|0>$ or in the state $\|1>$. Quantum mechanics is linear (the states, the kets, $\|>$ in the notation of Dirac, are vectors in a Hilbert space). If $\|0>$ and $\|1>$ are possible states of the qubit, $\frac{1}{\sqrt{2}}(\|0> + \|1>)$ is also a possible state. A qubit can be suspended in a quantum superposition between logic states. Similarly, the input register with n qubits a quantum computer can be prepared a quantum superposition of 2^n possible states, corresponding to 2^n numbers coded over n bits.
Structure of classical computer: Based on the realization of elementary gates.	**Structure of quantum computer:** Every quantum computing can be reduced to a series of manipulation of one or two qubits in quantum logic gates. As in classical logic, you only make a small number of elementary gates in order to then perform an arbitrary quantum computing by building a complex net work of gates.

Classical bit :	**Quantum bit:**
1 bit: 0 or 1. $\rightarrow$ Deterministic element: $b \in \{0, 1\}$. n bits code a value from $N = 2^n$. $\rightarrow$ To act on N amplitudes: N necessary operations. **n bits:** 0000........0 (0) 0000........1 (1) ... 1111........1 ($2^n - 1$) **State of bit:** $V = \alpha V_0 + \beta V_1$ with $\alpha + \beta = 1$, α and β reals. **Measure:** V, integrity of the state of bit.	1 bit: $\vert 0>$ or $\vert 1>$ or $\frac{1}{\sqrt{2}}(\vert 0> + \vert 1>)$. n bits can codes a superposition of 2^n values. $\rightarrow$ To act on N amplitudes: possible with 1 operation. **n qubits:** Quantum state of the form $\sum_{i=0}^{N-1} a_i \vert i>$; The information is contained in the amplitudes a_i associated with registers. **State of qubit:** $\vert \Psi >= \alpha \vert 0> + \beta \vert 1>$, with $\vert\alpha\vert^2 + \vert\beta\vert^2 = 1$, α et β complexes. **Measure:** Probabilistic orthogonal projection $\alpha \vert 0\rangle + \beta \vert 1\rangle \rightarrow Mesure \begin{matrix} \vert\alpha\vert^2 \\ \nearrow \\ \searrow \\ \vert\beta\vert^2 \end{matrix} \begin{matrix} \vert 0\rangle \\ \vert 1\rangle \end{matrix}$ **Evolution:** Unitary transformation (i.e, reversible): $G \in U(2)$ $G \in C^{2\times 2}$ such as $G^*G = Id$ $\vert\Psi >\bullet \cdots$ G 99K $\vert\Psi' >= G\vert\Psi >$ $\vert\Psi' >= G\vert\Psi >\bullet \cdots$ G* 99K$\vert\Psi >$

The space Fq :	**Hilbert space:**
Fq is a finite field with q elements, such as $q = p^n$ is power of a prime number.	Let (H, ||·||) a Hilbert space of finite dimension on C which is a vector space with the canonical inner product $\left\langle \sum_i z_i \vec{e}_i, \sum_j w_j \vec{f}_j \right\rangle = \sum_{i,j} \bar{z}_i w_j \left\langle \vec{e}_i, \vec{f}_j \right\rangle$ Where $\bar{z}$ denote the complex conjugate of z. • For a qubit which is the quantum version of a bit of information has a much larger number of states. These states are represented by an arrow that points a position on the surface of a sphere. → For a qūbit, the Hilbert space of dimension 2 on C, with the norm ||·||2. $\|0> = \binom{1}{0} \in$ H We notice $\|1> = \binom{0}{1} \in$ H ($\|0>, \|1>$) form an orthonormal base (H, ||·||2).
Classical linear code: A code is an injective mapping : $\{0, 1\}^k \rightarrow \{0,1\}^n$ such as: code ≠ codeword → Because all the codewords form a code. A binary code C is linear if the sum of two any codewords is another codeword: $\forall w1, w2 \in C, \quad w1+w2 \in C.$ A linear code is a sub vector space of $(Z/2Z)^n$. In particular the number of codewords is 2^k, where k is the dimension of C.	**Quantum stabilizer code:** Stabilizers codes are under a very rich family in quantum error correcting. They have this remarkable quality to be fully described by a discrete group: the stabilizer group of the code. This is a subgroup of Pauli group. → Let S a subgroup commutative of P_n such as $-1P_n \notin S$, C is a stabilizer code associated with the set S if: $C=\{\|\Psi> \in H^{\otimes n} / M\|\Psi> = \|\Psi> \forall M \in S\}$ $C \subseteq H$ (which is of dimension2^k).
Notation: $[n, k, d]$ n : the length k : the dimension d : the minimum distance	**Notation:** $[[n, k, d]]$ The double brackets used for quantum code distinguishes classical codes. n : the length k : the dimension d : the minimum distance
The locality (separability): The classical bits can be copied and multiplied as many times as you want.	**Quantum non-locality:** The no-cloning theorem shows that it is impossible to design a quantum cloned that can clone so i.e., that can perform the operation.

Type of errors: We can say that the errors that appear on a classic bit of type: bit inversion.	**Type of errors:** We can then say that the errors that appear on a qubit come from precisely of the Pauli group, i.e, it can three types of errors occur: - bit flip - phase flip - bit & phase flip • bit flip: $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; $X\|a\rangle = \|a \oplus 1\rangle$ • phase flip: $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$; $Z\|a\rangle = (-1)^a \|a\rangle$ • bit & phase flip: $Y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$; $Y\|a\rangle = i(-1)^a \|a \oplus 1\rangle$
The redondance: Classical correcting codes using redundancy to protect information.	**Intrication:** Quantum correcting codes used entanglement to relocate several physical systems the information encoded.
Internal law: The addition.	**Internal law:** The multiplication.
Parity matrix: $n - k$ rows, n columns.	**Stabilizer:** When we speak of stabilizers Group (associated with a code C), it is assumed that S satisfies the conditions: S is a commutative subgroup of P_n, such as $-1P_n \notin S$. $\rightarrow$ The elements of S are the stabilizer of C. $\rightarrow n - k$ generators of P_n commuting.
The distance: The Hamming distance, in the binary case $F2$ between two vectors X and Y of dimension n $(X, Y \in F_2{}^n)$ match the number of components for which the two vectors are different $d_H(X, Y) = \|\{i: X_i \neq Y_i,\ 0 \leq i \leq n\}\|$	**The distance:** In a similar manner of code classic.

The weight:	**The weight:**
The Hamming weight $W_H(X)$ of a vector X of F^n is the number of non-zero components of X. It is immediate that $\forall X \in F_2{}^n$, $W_H(X) = d_H(X,\underline{0})$ $W_H(X) = \#\{i \in \mathrm{N} / X_i \neq 0\}$.	The weight of a Pauli error is defined by $W(E) = \#\{i / E_i \neq I\}$.
Classical logic gates: Function "*Et*" Function "*NON*" Function "*NAND*" Function "*OU exclusive*".	**Quantum logic gates:** Gates on one qubit: - The gate "Not" *X* - The gate *Y* - The gate *Z* - The gate S. - The gate of Hadamard Gates on two qubits: - Controlled phase gate(*CZ*) - The gate CNOT Gates on three qubits: - Gate of Toffoli Controlled controlled not ≡ gate of Toffoli.

4. Examples of quantum error correcting codes

4.1. The five qubit code [[5, 1, 3]]

The $[[n, k, d]] = [[5, 1, 3]]$ perfect code encodes a single qubit $(k = 1)$, and corrects all errors of weight 1 (since $(d-1)/2 = 1$). This code is the smallest single-error correcting quantum code [4,5,6,7]. The cardinality of $[[5, 1, 3]]$ is $|S|=2^{n-k}=16$ and the set $D_S{}^{[[5,1,3]]}$ of $n-k=4$ stabilizer group generators is given by

$$D_S^{[[5,1,3]]} =< \sigma_x\sigma_z\sigma_z\sigma_x I, I\sigma_x\sigma_z\sigma_z\sigma_x, \sigma_x I\sigma_x\sigma_z\sigma_z, \sigma_z\sigma_x I\sigma_x\sigma_z >. \quad (3)$$

The code is the simultaneous eigenspace with eigenvalue 1 of 4 commuting check operators (stabilizer generators)

$$\begin{aligned} N_1 &= \sigma_x\sigma_z\sigma_z\sigma_x I, \\ N_2 &= I\sigma_x\sigma_z\sigma_z\sigma_x, \\ N_3 &= \sigma_x I\sigma_x\sigma_z\sigma_z, \\ N_4 &= \sigma_z\sigma_x I\sigma_x\sigma_z. \end{aligned} \quad (4)$$

All of these stabilizer generators square to I, they are mutually commuting because there are two collisions between σ_x and σ_z. The stabilizer and generator are given by

$$\begin{pmatrix} \sigma_x\sigma_z\sigma_z\sigma_x I \\ I\sigma_x\sigma_z\sigma_z\sigma_x \\ \sigma_x I\sigma_x\sigma_z\sigma_z \\ \sigma_z\sigma_x I\sigma_x\sigma_z \\ \sigma_x\sigma_x\sigma_x\sigma_x\sigma_x \\ \sigma_z\sigma_z\sigma_z\sigma_z\sigma_z \end{pmatrix}, \quad (5)$$

and the check parity matrix is

$$\mathrm{H}=\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & | & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & | & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & | & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & | & 1 & 0 & 1 & 0 & 0 \end{pmatrix}. \qquad (6)$$

We can take the basis codewords for this code to be

$$|0_L>=\sum_{N\epsilon D_S} N|00000>, \qquad (7)$$

and

$$|1_L>=\bar{X}|0_L>, \qquad (8)$$

that is

$$|0_L>=\frac{1}{4}[|00000>+|10010>+|01001>+|01010>-|11011>- |00100>-|11000>-|11101>-|00011>-|11110>- |01111>-|10001>-|01100>+|10111>+|00101> +|10100>] \qquad (9)$$

and

$$|1_L>=\frac{1}{4}[|11111>+|01101>+|10110>+|01011>+|10101>- |00100>-|11001>-|00111>-|00010>-|11100>- |00001>-|10000>-|01110>-|10011>-|01000> +|11010>]. \qquad (10)$$

4.2 Six qubit code [[6, 1, 3]]

The [[6, 1, 3]] code is the degenerate single-error correcting quantum code. Both the six-qubit code and the five-qubit code correct an arbitrary single-qubit error. But the six-qubit code has the advantage that it corrects a larger set of errors than the five-qubit code. This error correcting capability comes at the expense of a larger number of qubits - it corrects a larger set of errors because the Hilbert space for encoding is larger than that for the five-qubit code.

The cardinality of its stabilizer group S is |S| = 2^{n-k} = 32 and the set $U_S^{[[6,1,3]]}$ on n-k=5 stabilizer group generators is given by

$$U_S^{[[6,1,3]]}=<\sigma_y I\sigma_z\sigma_x\sigma_x\sigma_y, \sigma_z\sigma_x II\sigma_x\sigma_z, I\sigma_z\sigma_x\sigma_x\sigma_x\sigma_x, III\sigma_z I\sigma_z, \sigma_z\sigma_z\sigma_z I\sigma_z I>. \quad (11)$$

m_1	$\sigma_y I\sigma_z\sigma_x\sigma_x\sigma_y$
m_2	$\sigma_z\sigma_x II\sigma_x\sigma_z$
m_3	$I\sigma_z\sigma_x\sigma_x\sigma_x\sigma_x$
m_4	$III\sigma_z I\sigma_z$
m_5	$\sigma_z\sigma_z\sigma_z I\sigma_z I$
$\bar{X}$	$\sigma_z I\sigma_x I\sigma_x I$
$\bar{Z}$	$I\sigma_z II\sigma_z\sigma_z$

Table 2: The table presents the stabilizer of [[6,1,3]]

At least one generator from the six-qubit stabilizer anti-commutes with each of the single-qubit Pauli errors, X_i, Y_i, Z_i, where $i= 1,...,6$, because the generators have at least one Z and one X operator in all six positions. Additionally, at least one generator from the stabilizer anti-commutes with each pair of two distinct Pauli errors (except Z^4Z^6, which is in the stabilizer $U_S^{[[6,1,3]]}$). The check parity matrix is

$$\mathrm{H} = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \vert & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \qquad (12)$$

We can take the basis codewords for this code to be

$$|0_L>=\frac{1}{\sqrt{8}}[|000000>-|100111>+|001111>-|101000>+|010010>+ |110101>-|011101>-|111010>], \qquad (13)$$

and

$$|1_L>=\frac{1}{\sqrt{8}}[|001010>-|101101>+|000101>+|100010>-|011000>- |111111>-|010111>+|110000>]. \qquad (14)$$

4.3 CSS seven qubit code

The $[[n, k, d]] = [[7, 1, 3]]$ perfect code encodes a single qubit ($k = 1$), and corrects all errors of weight 1 (Since $(d - 1)/2 =1$). In order to construct a *CSS* (Calderbank–Shor–Steane) code you need to have two classical linear codes $C1$ $[n,k1]$, and $C2$ $[n,k2]$ such that $C2 \subset C1$ and $C1$, $C^{\perp}$ both correct errors. The simplest *CSS* code is the 7-bit code discovered by Steane. It uses the *CSS* construction with $H1 = H2 = H$. For CSS code:

$$\mathrm{H} = \begin{bmatrix} C_2 & 0 \\ 0 & C_1 \end{bmatrix} = [\ \mathrm{H}_Z, \mathrm{H}_X], \qquad (15)$$

this is the [7, 4, 3] Hamming code, it is single errors correcting and contains its dual, so leads to a single-error correcting quantum code of parameters $[[n, k, d]] = [[7, 1, 3]]$, i.e., a single qubit stored in 7, with minimum distance 3. The two codewords are:

$$|0_L>=\frac{1}{\sqrt{8}}[|0000000>+|1010101>+|00110011>+|101000>+|0001111>+ |1011010>+|0111100>0|1101001>], \qquad (16)$$

and

$$|1_L>=\frac{1}{\sqrt{8}}[|1111111>+|1001100>+|0101010>+|0011001>+|1110000>+ |1000011>+|0100101>+|0010110>]. \qquad (17)$$

The cardinality of $[[7, 1, 3]]$ is $|S| = 2^{n-k}= 64$ and the set $W_S^{[[7,1,3]]}$ of n-k=6 stabilizer group generators is given by

$$W_S^{[[7,1,3]]} =< \sigma_x\sigma_x\sigma_x\sigma_x III, \sigma_x\sigma_x II\sigma_x\sigma_x I, \sigma_x I\sigma_x I\sigma_x I\sigma_x, \sigma_z\sigma_z\sigma_z\sigma_z III, \sigma_z\sigma_z II\sigma_z\sigma_z I, \sigma_z I\sigma_z I\sigma_z I\sigma_z >. \qquad (18)$$

5. Codeword stabilized (*CWS*) codes

Codeword stabilized (*CWS*) codes [15,16,17] are a broad class of quantum error-correcting codes that include both additive and non additive quantum codes. Stabilizer codes can be considered a subset of *CWS* codes (though generally not in standard form).

Non-additive codes and additive codes have a difference in the dimension of the code space. An additive (stabilizer) code encodes a definite number k of logical qubits into a codeword of n physical qubits. Such a code with minimum distance d is denoted an $[[n, k, d]]$ code. The dimension of the code space is $K= 2^k$. For a non-additive code, the dimension K of the code space need not be a power of 2. Thus we introduce a different notation for non-additive codes; we denote a non-additive quantum code that encodes a K-dimensional code space into n physical qubits with minimum distance d as an $((n,K,d))$ code.

A *CWS* code is locally Clifford equivalent to a form specified by a graph *G* and a classical binary code. This is called standard form. In standard form, the graph *G* and its adjacency matrix *A* determines the word stabilizer of the *CWS* code. The graph *G* has *n* vertices, one for each qubit of the codeword. The word stabilizer S is a maximal Abelian subgroup of the Pauli group P_n, and has a set of generators corresponding to the vertices of the graph *G*. For a *CWS* code in standard form, the codeword stabilizer generators $\{S_i\}$ have the following structure:

$$S_i= X_i Z^{r_i}, \quad (19)$$

where r_i is the i-th row vector of the adjacency matrix *A*. That is, each generator S_i has a Pauli *X* operator on the qubit corresponding to vertex *i* of the graph, Pauli *Z* operators on the qubits corresponding to each of the neighbors of *i*, and identity operators I on all the other qubits. The word stabilizer $\boldsymbol{S}$ is generated by the set $\{S_i\}$.

A unique base state $|S>$. is the common +1 eigenstate of the word stabilizer S specified by the graph *G*. This state is fixed by any element $S \in \boldsymbol{S}$ of the word stabilizer:

$$|S>= S\,|S>.$$

The word operators $\{\omega_l\}$ are also elements of P_n. The code space is spanned by basis states obtained by applying word operators to the base state $|S>$, and each basis state is of the form

$$|\omega_l|>= \omega_l|S>.$$

Therefore, the number of the word operators determines the dimension of the code space, and the word operators map the base state onto an orthogonal state.

Theorem: An $[n, k]$ stabilizer code with stabilizer generators $S_1, ..., S_{n-k}$ and logical operations $\overline{X}_1 \ldots \overline{X}_k$ and $\overline{Z}_1 \ldots \overline{Z}_k$, is equivalent to the *CWS* code defined by

$$S = \dot{} S_1, ..., S_{n-k}, \overline{Z}_1 \ldots \overline{Z}_k$$

and word operators

$$\omega_v = \overline{X}_1{}^{(v)_1} \otimes \ldots \otimes \overline{X}_k{}^{(v)_k}$$

where v *is a* k-bit string.

6. RESULTS AND DISCUSSION

able 3: The table presents the correspondence between the stabilizer codes and CWS codes.

Stabilizer code	*CWS* code
Quantum error correcting code	Quantum error correcting code
Include just the additive code	Include the additive and non-additive code
Stabilizer code would have all + 1 eigenvalues for all codewords	The codewords $\omega_l \|S>$ are all eigenstates of All $s \in S$: $S\omega_l \|S> = \pm\omega_l \|S>$
For a general stabilizer code with basis vectors $\|\psi_i)$ to detect errors from a set ξ: $<\psi_i\|E\|\psi_j> = C_E\,\delta_{ij}$ for all $E \in \xi$	For codewords of the form $\|\omega_l> = \omega_l \|S>$ $< S\|\omega^{\dagger}_i E\omega_j\|S> = C_E\,\delta_{ij}$.
The stabilizer is $S = <S_1, \ldots, S_{n-k}>$, with logical operators $\overline{X}_k \ldots \overline{X}_{n-k}, \overline{Z}_k \ldots \overline{Z}_{n-k}$, define a stabilizer code.	The stabilizer is $S = S_1, \ldots, S_{n-k}, \overline{X}_1 \ldots \overline{X}_{n-k}$ and word operators $\overline{Z}_1 \ldots \overline{Z}_{n-k}$ define a CWS code.

7. CONCLUSION

Quantum error correction (*QEC*) plays an important role in quantum information processing and communication. Without *QEC* it is impossible to maintain a quantum state against the corrupting effects of decoherence for long enough to carry out nontrivial quantum computations or communication protocols.

In this paper we studied the correspondence between classical error correcting codes and the quantum error correcting codes precisely the stabilizer codes. We studied also the correspondence between the quantum stabilizer codes and CWS codes. For major conclusion, the CWS codes which contains the stabilizers codes and non-stabilizers code is more effective and more efficient compared to the stabilizers codes.

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