ORIGINAL ARTICLE

NUMERICAL INVESTIGATION OF TRANSIENT HEAT AND MASS TRANSFER OF MICRO POLAR FLUID OVER A VERTICAL POROUS SURFACE IN PRESENCE OF RADIATION AND CHEMICAL REACTION

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ABSTRACT

Background: Heat and mass transfer of micro polar fluid over a vertical porous surface in presence of radiation and chemical reaction has been discussed here. **Objective:** A model is arisen with boundary layer conditions and developed it by using mathematical methods. **Methods:** The dimensionless equations are solved by explicit finite difference method. **Results:** We have been determined the profiles of velocity, temperature, concentration and micro rotation for different values of some parameters. **Conclusion:** The effects of various parameters have shown graphically and an explanation is given at the end.
***Keywords**: Micro polar fluid, Porous surface, Micro Radiation, Chemical reaction.*

1. INTRODUCTION

The study of flow and heat transfer for an electrically conducting fluid past a plate has attracted the interest of many investigators in view of its applications in many engineering problems such as oil exploration, geothermal energy extractions and the boundary layer control in aerodynamics [1,2,3,4]. Specifically, Soundalgekar [1] obtained approximate solutions for the two dimensional how of an incompressible, viscous fluid flow past an infinite porous vertical plate with constant suction velocity normal to the plate. He found that the difference between the temperature of the plate and the free stream is significant to cause the free convection currents. Kim (2001) studied the unsteady free convection now of a micropolar fluid through a porous medium bounded by an infinite vertical plate [3]. Raptis (1998) studied numerically the case of a steady two-dimensional flow of a micropolar fluid past a continuously moving plate with a constant velocity in the presence of thermal radiation [4]. Gorla and Tornabene (1988) investigated the effects of thermal radiation on mixed convection flow over a vertical plate with non-uniform heat flux boundary conditions [5].

0n the other hand, heat transfer by simultaneous free or mixed convection and thermal radiation in the case of a micropolar fluid has not received as much attention. This is unfortunate because thermal radiation plays an important role in determining the overall surface heat transfer in situations where convective heat transfer coefficients are small. Such situations are common in space technology [6].

In the present work we consider the case of mixed convection how of a micropolar fluid past a semi-infinite, steadily moving porous plate with varying suction velocity normal to the plate in the presence of thermal radiation.

Micropolar fluids are fluids with microstructure belonging to a class of fluids with asymmetrical stress tensor. Physically, they represent fluids consisting of randomly oriented particles suspended in a viscous medium [7,8,9,10]. The micropolar fluid considered here is a gray, absorbing-emitting but non-scattering optically thick medium. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. It is assumed that the porous plate moves with constant upward velocity.

2. MATERIELS AND METHODES

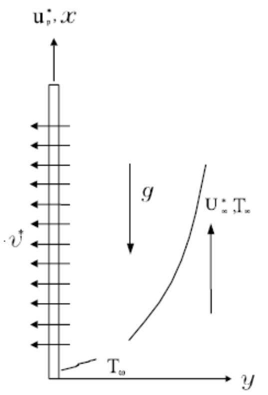

Figure 3.1: Physical Model and coordinate system of the problem.

Let us consider a two dimensional, unsteady flow of a laminar, incompressible micropolar fluid part a semi-infinite, vertical porous plate moving steadily and subjected to a thermal radiation field. The physical model and geometrical coordinates are shown in Figure 3.1. The x^*-axis is taken along the vertical plate in an upward direction and y^* -axis is taken normal to the plate.

The acceleration of gravity g is in a direction opposite to x^*- coordinate. It is assumed here that the size of holes in the porous plate is much larger than a characteristic microscopic length scale of the micropolar fluid to simplify formulation to simplify formulation of the boundary conditions. Further, due to the semi-infinite plane surface assumption, the flow Variables are function of normal distance $y^*$and time t^*only.

Under the boundary layer approximation the governing equations for me transient heat and mass transfer of micropolar fluid over a porous surface in presence of radiation and chemical reaction can be expended as follows ;
The governing equation is:

Continuity equation:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

Momentum equation:

$$\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + (v + v_r)\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_f(T - T_\infty) + 2v_r\frac{\partial \omega^*}{\partial y^*} \tag{2}$$

Micro rotation equation:

$$\rho j^*\left(\frac{\partial \omega^*}{\partial t^*} + v^*\frac{\partial \omega^*}{\partial y^*}\right) = \gamma\frac{\partial^2 \omega^*}{\partial y^{*2}} \tag{3}$$

Energy equation:

$$\frac{\partial T}{\partial t^*} + v^*\frac{\partial T}{\partial y^*} = \alpha\left(\frac{\partial^2 T}{\partial y^{*2}} - \frac{1}{k}\frac{\partial qr}{\partial y^*}\right) \tag{4}$$

Concentration equation:

$$\frac{\partial C^*}{\partial t^*} + v^*\frac{\partial C^*}{\partial y^*} = D^*\frac{\partial^2 C^*}{\partial y^{*2}} - R^*(C^* - C^*_\infty) \tag{5}$$

By using Rossland approximation

$$q_r = -\frac{4}{3}\frac{\sigma_1}{k_1}\frac{\partial T^4}{\partial Y^*}$$

Where σ_1 is the Stefan-Boltzman constant and k_1 is the mean approximation coefficient
Expanding T^4 in Taylor Series

$$T^4 \cong T_\infty{}^4 + (T - T_\infty)\,4T_\infty{}^3$$

$$= 4TT_\infty{}^3 - 3T_\infty{}^4$$

$$q_r = \frac{-16.\sigma_1 T\infty^3}{3k_1}\frac{\partial T}{\partial y^*}$$

$$\therefore \frac{\partial qr}{\partial y^*} = \frac{-16.\sigma_1 T\infty^3}{3k_1}\frac{\partial^2 T}{\partial y^{*2}}$$

Again let

$$-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} = \frac{\partial u^*_\infty}{\partial t^*}$$

So the equation (1) to (5) because

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{6}$$

$$\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{\partial u\infty}{\partial t^*} + (v + vr)\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta f\,(T - T\infty) + 2vr\frac{\partial \omega^*}{\partial y^*} \tag{7}$$

$$\rho j^*\left(\frac{\partial \omega^*}{\partial t^*} + v^*\frac{\partial \omega^*}{\partial y^*}\right) = \gamma\frac{\partial^2 \omega^*}{\partial y^{*2}} \tag{8}$$

$$\frac{\partial T}{\partial t^*} + v^*\frac{\partial T}{\partial y^*} = \alpha\left(\frac{\partial^2 T}{\partial y^{*2}} + \frac{16\sigma 1T\infty^3}{3KK1}\frac{\partial^2 T}{\partial y^{*2}}\right) \tag{9}$$

$$\frac{\partial C^*}{\partial t^*} + v^*\frac{\partial C^*}{\partial y^*} = D^*\frac{\partial^2 C}{\partial y^{*2}} - R^*\,(C^* - C^*\infty) \tag{10}$$

The Corresponding initial and boundary condition because:

$$u^* = u^*_p\,,\; v^* = -V_0\,, T = T\omega\,, C^* = C\omega$$

$$\omega^* = -n\frac{\partial u^*}{\partial y^*} \text{ at } y^* = 0$$
$$u^* = 0, v^* = 0, T = T\infty, \quad c = c_\infty, \omega = 0 \; at y^* \to \infty \quad (11)$$

Here $u^* v^*$ are the velocity components along x^* and y^* directions, respectively, v is the kinematic viscosity, v_r is the kinematic viscosity, β_r is the coefficient of volumetric thermal expansion of the fluid, T is the temperature, α is the effective thermal diffusivity of the fluid, and k is the effective thermal conductivity.

Since the governing equations inch ding the initial and boundary conditions will solved based on an explicit finite difference method. Therefore, it is required the governing equations to be a dimensionless form.

In order to dimensionalize the above equation, we introduce the following dimensionalize variable:

$$U = \frac{u^*}{U_0}; V = \frac{v^*}{U_0}; Y = \frac{V_0}{v}y^*$$
$$U_\infty = \frac{U_\infty^*}{U_0}; Up = \frac{u_p^*}{U_0}; W = \frac{v}{U_0 V_0}\omega^*$$
$$\tau = \frac{V_0^2}{v}t^*; \theta = \frac{T - T_\infty}{T\omega - T_\infty}; \delta = \frac{v}{V_0^2}\delta^*$$
$$J = \frac{V_0^2}{v^2}j^*; C = \frac{c^* - c_\infty^*}{c_\omega^* - c_\infty^*}$$
$$\xi = \frac{R^* v}{V_0^2}; Sc = \frac{v}{D^*}; \beta = \frac{v_r}{v}$$
$$\lambda = \frac{\mu j^*}{\gamma}, \Gamma = \left(1 - \frac{4}{3R+4}\right)p_r$$

Where

S_c = Schmidt number
ξ = chemical reaction parameter
β = Dimensionless Viscosity ratio
$Pr = \frac{v}{\alpha}$ = Prandtl number
$R = \frac{KK1}{4\sigma T\infty 3}$ = Radiation parameter
Γ = Leat transfer coefficient

Using these dimensionless Variables equation (6) to (11) becomes

$$\frac{\partial V}{\partial Y} = 0 \quad (12)$$
$$\frac{\partial U}{\partial \tau} - \frac{\partial U}{\partial Y} = \frac{dU\infty}{d\tau} + (1+\beta)\frac{\partial^2 U}{\partial Y^2} + Gr\theta + 2\beta\frac{\partial \omega}{\partial Y} \quad (13)$$
$$\frac{\partial \omega}{\partial \tau} - \frac{\partial \omega}{\partial Y} = \frac{1}{\lambda}\frac{\partial^2 \omega}{\partial Y^2} \quad (14)$$
$$\frac{\partial \theta}{\partial \tau} - \frac{\partial \theta}{\partial Y} = \frac{1}{\Gamma}\frac{\partial^2 \theta}{\partial Y^2} \quad (15)$$
$$\frac{\partial c}{\partial \tau} - \frac{\partial c}{\partial Y} = \frac{1}{Sc}\frac{\partial^2}{\partial Y^2} - \xi C \quad (16)$$

The boundary condition becomes

$$U = Up, V = f\omega, \theta = 1, \omega = -n\frac{du}{dY}$$
$$C = 1 \; at \; Y = 0$$
$$U = 0, \theta = 0, \omega = 0, C = 0 \; at \; Y \to \infty$$

3. Numerical Solutions:

In order to solve a non-dimensional system by the explicit finite difference method, it is required a set of finite difference equations. In this case, the region within the boundary layer is divided by some mesh of lines parallel to X and Y axes where X – axis is taken along the plate and Y – axis is normal to the plate.

Here, we regard Y max (= 35) as corresponding to Y→ ∞ i,e Y varies from 0 to 35 have been considered. Consider n = 100 in Y- direction 9t is assumed that ΔY are constant mesh sizes along Y- dissection and taken as follows ΔY = 0.2 ($0 \leq Y \leq 35$) with the smaller time step $\Delta\tau$ = 0.0005.

LetU', V', ω', θ', C' denote the Values of U, V, $\omega, \theta,$ C at the end of a time step respectively.
Using the explicit finite difference approximation we obtain the following appropriate set of finite difference equations:

Continuity:

$$\frac{Vi,j - Vi,j-1}{\Delta Y} = 0$$

Momentum:

$$\frac{U_{i}^{/},j-Ui,j}{\Delta\tau} - \frac{Ui,j+1-Ui,j}{\Delta Y} = \frac{du\infty}{d\tau} + \frac{(1+\beta)Ui,j+1-2Ui,j+Ui,j-1}{(\Delta Y)^2} + Gr\ Ti,j + 2\beta\frac{\omega i,j+1-\omega i,j}{\Delta Y}$$

Micro rotation equation:

$$\frac{\omega i^{/},j-\omega i,j}{\Delta\tau} - \frac{\omega i,j+1-\omega i,j}{\Delta Y} = \frac{1}{\lambda}\frac{\omega i,j+1-2\omega i,j+\omega i,j-1}{(\Delta Y)^2}$$

Energy equation:

$$\frac{Ti^{/},j-Ti,j}{\Delta\tau} - \frac{Ti,j+1-Ti,j}{\Delta Y} = \frac{1}{\Gamma}\frac{Ti,j+1-2Ti,j+Ti,j-1}{(\Delta Y)^2}$$

Concentration equation:

$$\frac{Ci',j-Ci,j}{\Delta\tau} - \frac{Ci,j+1-Ci,j}{\Delta Y} = \frac{1}{Sc}\frac{Ci,j+1-2Ci,j+Ci,j-1}{(\Delta Y)^2} \ldots\ldots\xi Ci,$$

4. RESULTS AND DISCUSSIONS

The formulation of the problem that accounts for the effect of radiation field on the flow and heat transfer of an incompressible micro-polar fluid along a semi-infinite moving vertical porous plate was accomplished out in the preceding sections. This enables us to carry out the numerical computations for the velocity, micro-rotation and temperature fields for various values of the flow conditions and fluid properties. In the calculations, the boundary condition for $y \to \infty$ is replaced by $y = y_{max}$, where y_{max} is a suiliciently large value of the distance away from the plate where the velocity profiles u approaches a given free stream velocity. We chose $y_{max} = 35$ and a step size $y = 0.35$.

Figures 3.2-3.17 show representative plots of the stream wise velocity, concentration and micro rotations well as temperature profiles for a micro-polar fluid with the fixed flow conditions a$\tau = 0.0005$ where $S_c = 0.06$, $\xi = 0.5$, $\beta = 0.1$, $P_r = 0.7$, $R = 1.0$, $\lambda = 0.2$, $U_p = 0.5$, $S = 0.5$, which are listed in the figure legend.

Figure 3.2 and 3.3 express the effect of prandtl number (Pr) on velocity and temperature profiles respectively. From Figure 3.2 we see that the velocity decreases rapidly with the increase of Pr. On the other hand from the Figure 3.3 we see that the temperature profiles increase with the increase of Pr.

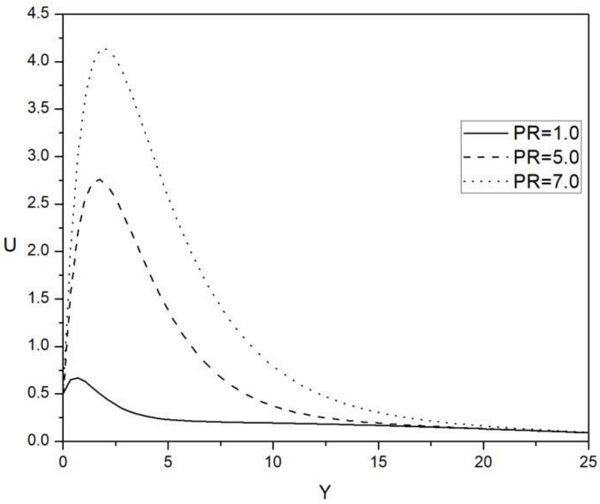

Figure 3.2: Velocity profiles for different values of prandtl number.

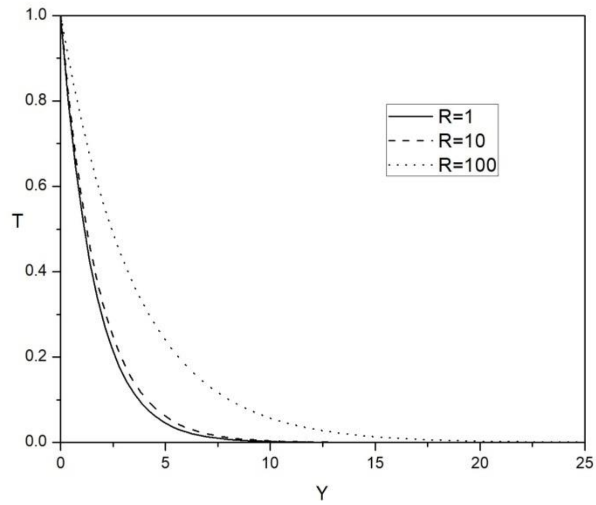

Figure 3.5: Temperature profiles for different values of Radiation parameter.

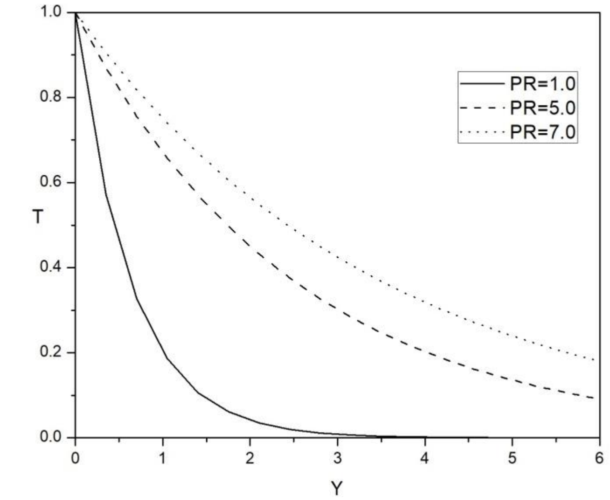

Figure 3.3: Temperature profiles for different values of prandtl number.

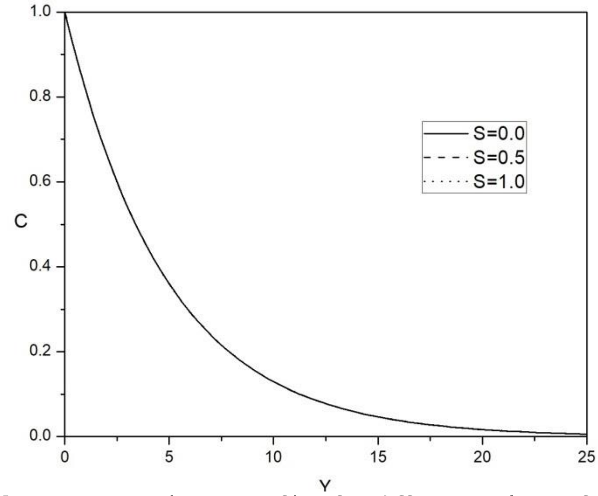

Figure 3.6: Velocity profiles for different values of micro-gyration boundary condition.

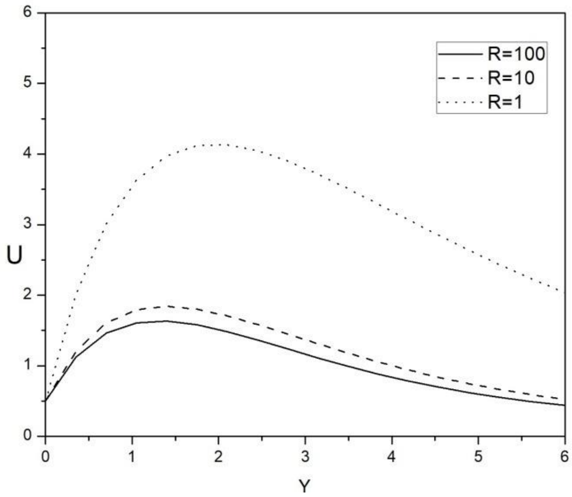

Figure 3.4: Velocity profiles for different values of Radiation parameter.

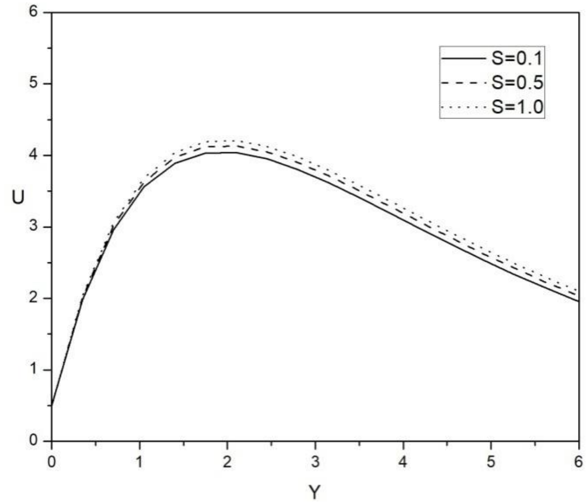

Figure 3.7: Concentration profiles for different values parameter of micro-gyration boundary condition.

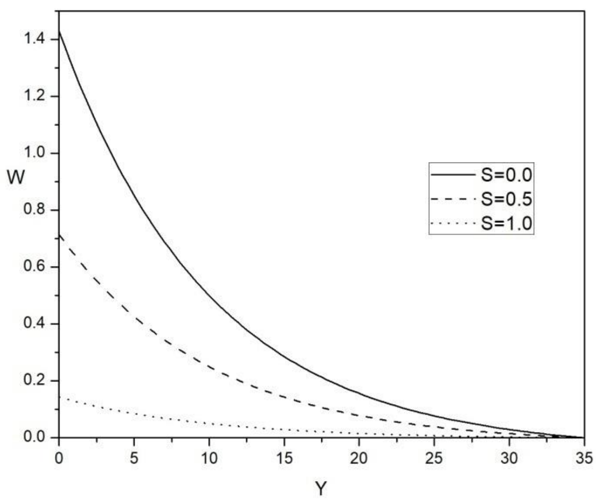

Figure 3.8: Micro Rotation profiles for different values parameter of micro-gyration boundary condition.

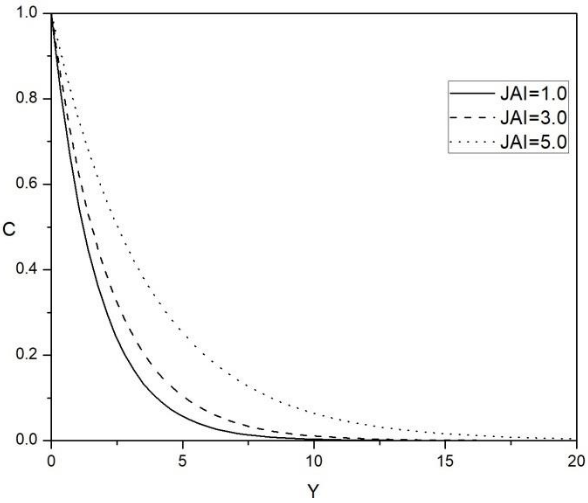

Figure 3.11: Concentration profiles for different values of Chemical Reaction parameter.

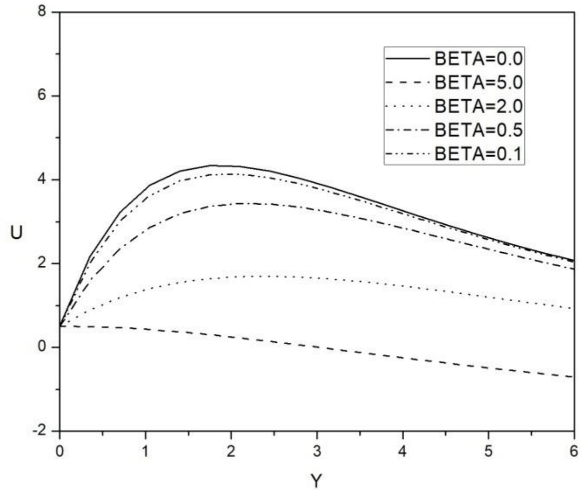

Figure 3.9: Velocity Profiles for different values of dimensionless viscosity ratio

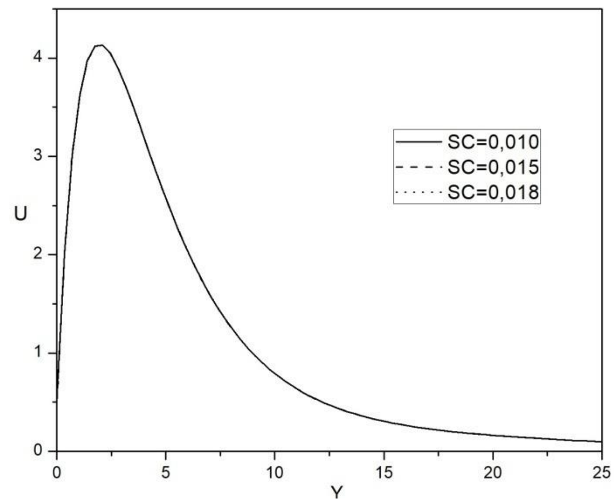

Figure 3.12: Velocity profiles for different values of Schmidt number

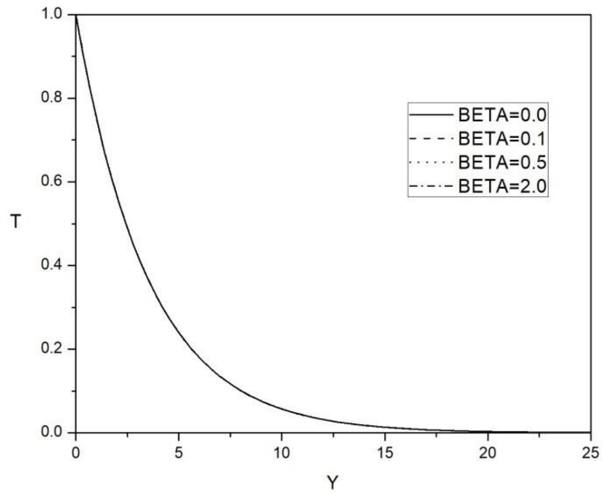

Figure 3.10: Temperature profiles for different values of dimensionless viscosity ratio

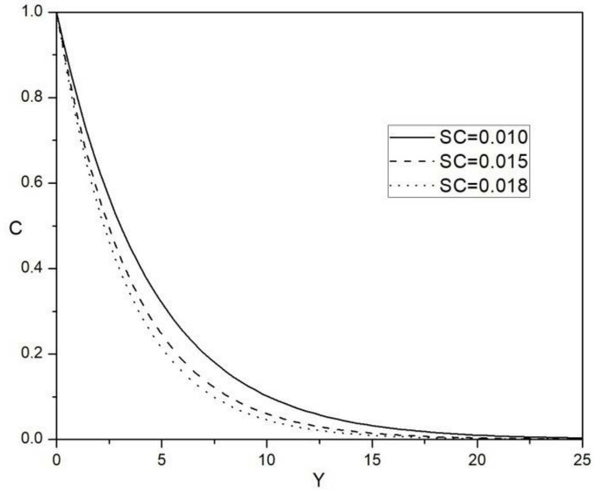

Figure 3.13: Concentration profiles for different values of Schmidt number

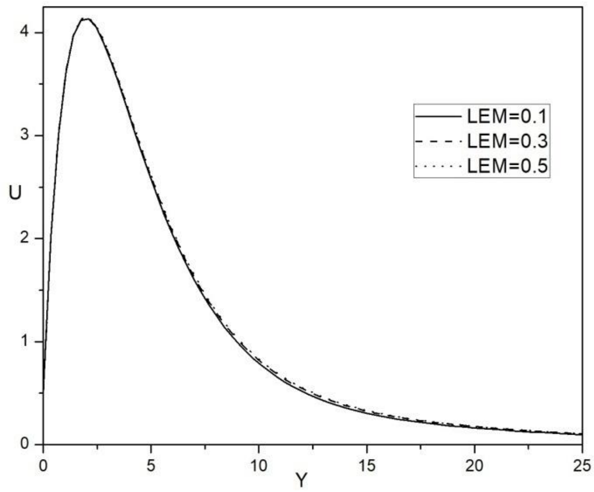

Figure 3.11: Velocity profiles for different values of dimensionless materials parameter.

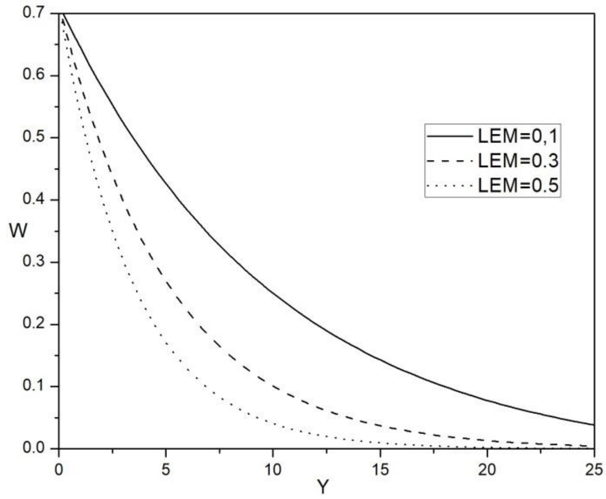

Figure 3.14: Micro rotation profiles for different values of dimensionless materials parameter.

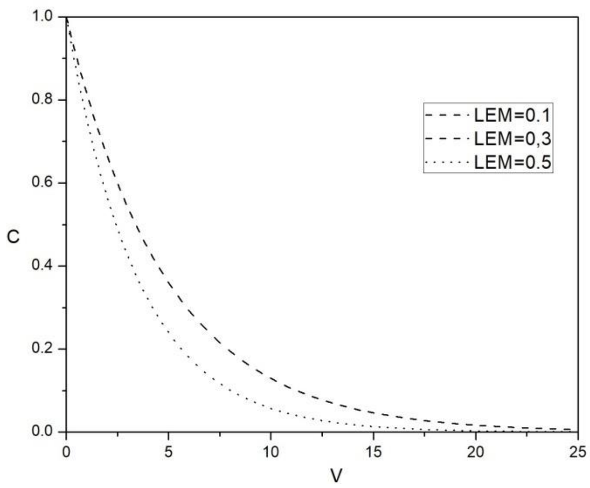

Figure 3.15: Concentration profiles for different values of dimensionless materials parameter.

Figure 3.4 and 3.5 express the effect of Radiation parameter (R) on velocity and temperature profiles respectively. From Figure 3.4 we see that the velocity decreases rapidly with the increase of R. On the other hand from the Figure 3.5 we see that the temperature profiles increase with the increase of R.

Figure 3.6, 3.7 and 3.8 express the effect of micro-gyration boundary condition parameter (S) on velocity, concentration and micro rotation profiles respectively. In the Figure 3.6 we see that the velocity increases rapidly with the increase of S. and from the Figure 3.7 we see that the concentration profiles decrease with the increase of S. On the other hand Figure 3.8 represent that micro rotation profiles decrees with the increase of micro gyration boundary condition parameter S.

Effect of viscosity ratio parameter (β) on velocity and temperature profiles show in Figure 3.9 and Figure 3.10. In these Figures we see that the velocity and temperature profile decreases with the increase of viscosity ratio parameter β.

The concentration profiles with respect to span wise coordinate Y for various values of chemical reaction parameter ξ is shown in Figure 3.11. the result show that as an increasing of the chemical reaction parameter ξ the concentration profile decrees.

Figure 3.12 and 3.13 express the effect of Schmidt number (Sr) on velocity and concentration profiles respectively. From Figure 31.2 we see that the velocity increases with the increase of different values of Sc. On the other hand from the Figure 3.13 we see that the concentration profiles decrease with the increase of Sc.

The velocity, concentration and micro rotation profiles with respect to span wise coordinate Y for various values of dimensionless materials parameter λ is shown in Figure 3.14, 3.15 and 3.16 respectively. In the Figure 3.14 we see that the velocity increase with the increase of different values of λ . On the other hand we understand from Figure 3.15 and 3.16 the concentration of micro rotation profiles decrees with increase of dimensionless materials parameter λ .

The velocity profiles with respect to span wise coordinate Y for various values of Grashop number (Gr) is shown in Figure 3.17. The result show that when an increasing in the Grashop number (Gr) the velocity profile is increasing.

5. CONCLUSION

The transient heat and mass transfer of micropolar fluid over a vertical porous surface in presence of radiation and chemical reaction is studied. The governing equations are simplified by using a set of dimensionless variable and then solved numerically using explicit finite difference method.

We have examined the problem of an unsteady, incompressible mixed convection flow of micropolar fluid past a semi-infinite porous plate whose velocity is maintained constant in the presence of a radiation field. 'The method of solution has been developed in the limit of small perturbation approximation. Numerical results are presented to illustrate the details of the flow and heat transfer characteristics and their dependence on the how conditions and fluid properties. In particular, we found that in a radiation-dominated problem (i.e., radiation parameter R is small), thermal and momentum boundary layers increase in size, thereby leading to enhanced buoyancy-induced transport but decreased rate of heat transfer at the wall. We also found that there is an optimal value of radiation parameter that results in a minimum friction at the surface of the wall.

For better understanding of the fluid-mechanical and thermal behavior of this flow problem, however, it may be necessary to perform the experimental works. In the near future we would be glad to compare these analytical results with those obtained by anyone in the same field.

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